Energy Prices Volatility and the United Kingdom: Evidence from a Dynamic Stochastic General Equilibrium Model

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Abstract
This paper analyses the consequences and effects of volatile energy prices in the UK. The evidence provided are from an estimated DSGE model of energy. The model is applied on filtered data from 1981:Q1 to 2013:Q1 and evaluated by the indirect inference testing. In analysing the structural shocks, the study found higher volatility in energy prices shock during the Great Recession compared to the sample period. The high volatility of energy prices shock caused inflationary pressures in the economy. The study found energy prices shock amplified the Great Recession by significantly contributing to the fall in output. Thus, energy prices shock is an important driver of economic activity. However, given the shocks are stationary, energy prices shock is temporary. Therefore, all consequences of energy prices in the economy are short-term. By implication, when volatile energy prices create an output shortfall, monetary policy is the tool used to off-set short-term falls in output. We find results persists with robustness check. The findings justified why the DSGE model is a policymakers’ workhorse.

Key words: Business cycle; Energy prices; Great Recession; Output; Volatility
1.0 Introduction

Energy is one of the most important driving forces that shape the economic growth of industrialised economies. In production, energy is an input that compliments labour. Thus, like wages, energy prices affect overall production costs. The effect of volatile energy prices reflects on production costs, which, in turn, influences output. Recent studies have provided an insight into energy price volatility and economic activity. Kilian (2017), Holtemoeller and Mallick (2016) and Sisodia, et al. (2015) found that volatile energy prices could influence energy demand. For example, a reduction in energy demand due to higher prices may lead to a decline in output. A decline in output may require a governmental response, such as borrowing to bridge the loss of output. There is an ongoing debate surrounding monetary policy responses to the consequences of volatile energy prices (Bernanke, Gertler and Watson, 1997; Hamilton and Herrera, 2004; Kilian and Lewis 2010; Dixon, Franklin and Millard, 2014). Some debates discuss theoretical modelling, while some focus on empirical evidence and others consider both. Some also argue that energy prices are a source of economic fluctuation. The literature on volatile energy prices is constantly expanding.

The dynamic stochastic general equilibrium (DSGE) model is regarded to be the most transparent and open models used for macroeconomic analysis (Christiano, 2018). The openness of the DSGE model allows for multidisciplinary research. However, few studies have considered regime shifts and responses to volatile energy prices. Historically, such models were developed in response to the Lucas Critique. Lucas (1976) stated that parameters of econometric models were not deep enough to assess policy interventions. Since the development of this critique, further literature has augmented business cycle models with New Keynesian (NK) nominal rigidities\(^2\) to provide a standard stabilisation role for monetary policies.

Nakov and Pescatori (2010) incorporated a standard model of inflation using Bayesian estimation on U.S. data before the Great Recession. They found declining energy share output accounted for one-third of inflation. They concluded that the variation in inflation moderation could be explained using monetary policy. Harrison et al. (2011) analysed the effects of permanent energy price shocks on United Kingdom (UK) inflation as natural resources declined in the UK. They suggested that an extension of their work could be done to consider regime shifts in UK monetary policy. More recently, Aminu, Meenagh and Minford (2018) examined the effects of energy prices on the UK economy and observed that declining output

\(^2\) See Annicchiarico, Pelloni and Rossi (2011).
was mainly driven by volatility in energy prices during the Great Moderation. In particular, they found that the declining effect was mainly due to low demand in energy-intensive sectors. However, they did not account for the role of monetary policies in the economy.

The goal of this study is to account for the consequences and effects of volatile energy prices in the UK using evidence from a DSGE model. The objectives of this study are as follows: firstly, to estimate a DSGE model of energy using the indirect inference testing method; secondly, to study the importance of energy price shocks and how they drive the business cycle; and thirdly, to join the debate to support the view that monetary policy is able to reduce the consequences of world energy prices. Hamilton and Herrera (2004) opposes the view as initially raised by Bernanke et al. (1997). This study will show that when volatile energy prices create an output shortfall, monetary policy is the tool used to off-set short-term falls in output.

The rest of this paper is structured as follows: section 2 will present the model; section 3 will discuss the data and methodology used in the study; section 4 will evaluate the estimated model, while section 5 will analyse the shocks and section 6 will provide a conclusion.

2.0 Model

The model had two unique features, which set it apart from mainstream DSGE models. Firstly, the model had three consumption goods - two energy outputs (petrol and utilities) and one non-energy output. A combination of these gave the gross final output. Secondly, the model introduces supply chain movement of energy products - from world energy (oil and gas) to end products of petrol and utilities. Using the prices of each product, this study linked the prices of energy products to aggregate consumption prices. Figure 1 depicts a graphical representation of the model, a circular flow of income of an open economy, augmented from Aminu (2018).

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3 Non-energy output refers to all the goods and services produced in the economy that are not petrol products or utilities.
Figure 1 Circular Flow of Income with Energy Supply.

The economy starts transferring unused output to accumulate capital ($K$) using a utilisation rate $z$. The accumulated capital and labour ($h$) combine to produce intermediate good ($V$). $V$ is produced for the three sectors as: non-energy intermediate good ($V_n$), intermediate good of utilities ($V_u$), and intermediate good of petrol ($V_p$). In the petroleum sector, $V_p$ and crude oil ($O$) are combined to produce final petroleum output ($Y_p$). $O$ is the combination of extracted crude oil in the UK ($\bar{O}$) and imported crude oil ($X_O$). Production of utilities is similar to the petroleum sector, where $V_u$ and gas ($G$) is used to produce utilities output ($Y_u$). $G$ is the combination of extracted gas in the UK ($\bar{G}$) and imported gas ($X_g$). In the non-energy sector, $V_n$ and imported intermediate good ($M$) are combined to produce non-energy output ($Y$). The combination of the final output from all sectors gives the gross output ($Q$). Each sector, then, trades its output to households for consumption ($C$), to the government ($C_g$) and to the rest of the world as exports ($X$). The rest of the stock is reinvested which accumulates as capital.

The quantitative model is a log-linearized model. The equations that govern household’s decision, firm’s decision, the monetary policy, and the world market are presented.

**Household**

Consumers maximise their utility by choosing how much to consume, given their wealth and relative prices. Equation (1) determines the optimal choice of consumption by household

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4 The full version of the linearized model is available in Aminu (2018).
between one period and another. This is consistent in literature, as in Smets and Wouters (2007).

\[
\hat{c}_t = \frac{\psi_{hab}(1-\sigma_c)}{1+\psi_{hab}(1-\sigma_c)} \hat{c}_{t-1} + \frac{1}{1+\psi_{hab}(1-\sigma_c)} E_t \hat{c}_{t+1} - \frac{\sigma_c}{1+\psi_{hab}(1-\sigma_c)} \left( t_t - E_t \hat{h}_{c,t+1} - \left( \frac{1}{\beta} - 1 \right) + \epsilon_{b,t} \right) \tag{1}
\]

\( c \) denotes consumption of household. \( E \) denotes the rational expectation operator which satisfies the assumption of a forward-looking model. \( \epsilon_b \) denotes the preference shock to consumer demand. The real interest rate is derived by subtracting the nominal interest rate, \( i \), from the consumer inflation, \( \pi_c \). The parameter, \( \beta \) is the discount factor of intertemporal consumption. \( \psi_{hab} \) denotes the household’s habit persistence, while \( \sigma_c \) implies household’s elasticity of intertemporal substitution.

The model assumes that individuals decide whether to buy foreign bonds or to hold domestic bonds. Household’s decision depends on foreign interest rate. If rates are higher abroad, they will buy foreign bonds and vice-versa. Foreign bonds trade incur quadratic costs resulting to an uncovered interest parity condition:

\[
E_t \hat{s}_{t+1} - \hat{s}_t = - \left( \frac{1}{\beta} - 1 \right) + \chi_{bf} \hat{b}_{f,t} + \epsilon_{rf,t} \tag{2}
\]

where \( s \) represents the rate of foreign exchange. Foreign bond is denoted by \( b_f \). \( \epsilon_{rf} \) denotes the exogenous shock of world interest rates. The parameter \( \chi_{bf} \) denotes adjustment cost of household’s bond portfolio.

Households are assumed to be the only suppliers of labour. Thus, they supply labour in a differentiated manner - according to skill level and incentive to work. They set the economy’s real wages (\( w \)) based on a mark-up over marginal rate of substitution (\( mrs \)) between leisure and consumption, subject to wage inflation (\( \dot{W} \)):

\[
\dot{W}_t = \frac{\xi_w}{1 + \beta \xi_w} \dot{W}_{t-1} + \frac{\beta}{1 + \beta \xi_w} E_t \dot{W}_{t+1} - \left( \frac{\psi_w(1 - \beta(1 - \psi_w))}{\left( 1 + \sigma_w \sigma_n \right) \left( 1 - \psi_w \right) \left( 1 + \beta \xi_w \right)} \right) \left( \dot{W}_t - mrs_t \right) + \epsilon_{w,t} \tag{3}
\]

\( \epsilon_w \) denotes the wage mark-up exogenous shock. \( \xi_w \) denotes the parameter of wage indexation, while \( \psi_w \) denotes a probability of household’s ability to change the wage rate. The set wages are subject to wage stickiness.

**The firm**

**The energy sectors**

Two sectors that produce energy goods, petroleum and utilities. Both sectors are similar in terms of production and price-setting.

\(^5\text{Following the Fisher equation.}\)
**Petroleum sector**

Output is produced by a Leontief production function where factors of production are fixed with no factor substitutability. Intuitively, adding workers to a given amount of crude oil will not increase the output. Production in the petroleum sector gives the following equation:

\[
\hat{y}_{p,t} = \hat{l}_{o,t} = \hat{V}_{p,t} \tag{4}
\]

\[
\pi_{p,t} = \frac{\beta}{(1 + \beta \varepsilon_{pp})} E_t \pi_{p,t+1} + \frac{\varepsilon}{(1 + \beta \varepsilon_{pp})} \pi_{p,t-1} + \frac{(1 - \chi_{pp})(1 - \beta \chi_{pp})}{(1 + \beta \varepsilon_{pp}) \chi_{pp}} \hat{\sigma}_{p,t} \tag{5}
\]

where \(y_p\) is denoted as petrol output, \(\hat{l}_o\) is denoted as crude oil input to produce petrol. In price setting, the petroleum sector is subject to nominal rigidities which result in a New Keynesian Phillips Curve (NKPC) in (5). \(\pi_p\) denotes the inflation on petroleum prices. \(\hat{\sigma}_p\) represents firm’s marginal cost of producing petrol. The parameters, \(\varepsilon_{pp}\) denotes the firm’s degree of indexation. \(\chi_{pp}\) denotes firm’s probability of not changing its output price.

**Utilities sector**

Similarly, utilities are produced by a Leontief production function where factors of production are fixed with no factor substitutability. This gives:

\[
\hat{y}_{u,t} = \hat{l}_{g,t} = \hat{V}_{u,t} \tag{6}
\]

\[
\pi_{u,t} = \frac{\beta}{(1 + \beta \varepsilon_{uu})} E_t \pi_{u,t+1} + \frac{\varepsilon}{(1 + \beta \varepsilon_{uu})} \pi_{u,t-1} + \frac{(1 - \chi_{uu})(1 - \beta \chi_{uu})}{(1 + \beta \varepsilon_{uu}) \chi_{uu}} \hat{\sigma}_{u,t} \tag{7}
\]

where \(y_u\) denotes utility output, \(\hat{l}_g\) denotes gas input. The sector is subject to nominal rigidities which result in a NKPC in (7) when setting prices. \(\pi_u\) denotes the inflation on prices of utilities. \(\hat{\sigma}_u\) represents firm’s marginal cost of producing utilities. The parameters, \(\varepsilon_{uu}\) denotes the firm’s degree of indexation. \(\chi_{uu}\) denotes firm’s probability of not changing its output price.

**Non-energy sector**

However, firms that produce non-energy goods follow a Cobb-Douglass production function and displays constant returns to scale.

\[
\hat{y}_{n,t} = (1 - \alpha_y) k g_t + \alpha_y \hat{e}_t + \varepsilon_{a,t} \tag{8}
\]

where \(y\) denotes final output of non-energy. \(\varepsilon_{a}\) denotes non-energy output exogenous shock productivity shock. \(kg\) denotes combination of intermediate goods used in production. \(e\) denotes the energy input in production where firms have the choice to use petroleum or other utilities without any restriction as \(\hat{e}_t = \hat{l}_{p,t} = \hat{l}_{u,t}\). Similar schedules to (8) have also been
assumed by Harrison et al. (2011), Dixon et al. (2014) and Aminu (2018). The parameter $\alpha_y$ explains the cost share of energy use in the final output of non-energy sector.

Assuming sticky prices, firms’ price-setting is subject to nominal rigidities. The resulting NKPC is the price mark-up:

$$\hat{p}_t = \frac{\beta}{(1+\beta\varepsilon)}E_t\hat{p}_{t+1} + \frac{\varepsilon_p}{(1+\beta\varepsilon)}\hat{p}_{t-1} + \frac{(1-\chi_p)(1-\beta\chi_p)}{(1+\beta\varepsilon)\chi_p}\widehat{\sigma}_t + \varepsilon_{\mu t} \tag{9}$$

$\pi_t$ represents inflation rate, $\sigma$ denotes the marginal cost and $\varepsilon_{\mu}$ denotes the exogenous shock of price mark-up. $\varepsilon_p$ is a parameter that denotes the firm’s degree of indexation. $\chi_p$ represent the probability of firms’ inability to change the price. Thus, firms optimally set their prices with a probability of $1 - \chi_p$, in each period. However, if optimal price changing not achievable, a partial price indexation to lagged inflation will be done.

Combined output, from the three sectors, gives the gross output in the economy. Therefore the market clears with the following equation:

$$\pi_{c,t} = \frac{c_n}{p_c} \hat{p}_t + \frac{p_p c_p}{p_c} \pi_{u,t} + \left(1 - \frac{c_n}{p_c} - \frac{p_p c_p}{p_c}\right)\pi_{u,t} \tag{10}$$

$q_t = \frac{y_n}{y_t} \hat{y}_{n,t} + \frac{y_u}{y_t} \hat{y}_{p,t} + \left(1 + \frac{y_n}{y_t} - \frac{y_p}{y_t}\right)\hat{y}_{u,t} \tag{11}$

where $\frac{c_n}{p_c}$ and $\frac{p_p c_p}{p_c}$ denotes steady-state parameters of sectoral prices. $\frac{y_n}{y_t}$ and $\frac{y_u}{y_t}$ denotes steady-state parameters of output.

- **Fiscal and Monetary policy**

The fiscal authority ($G$) uses lumpsum taxes ($T$) on household to balance its budget. Its budget constraint is given by:

$$G_t = \psi_p p_{p,t} y_p + \psi_u p_{u,t} y_u + T_t \tag{12}$$

where $p_p$ and $p_u$ are the respective prices of petroleum and utilities. $\psi_p$ and $\psi_u$ are parameters denoting cost share of intermediate goods of petroleum and utilities.

The model assumes the Bank of England follows a Taylor rule monetary policy:

$$i_t - \left(\frac{1}{\beta} - 1\right) = \theta_{rg} (i_{t-1} - \left(\frac{1}{\beta} - 1\right)) + (1 - \theta_{rg})(\theta_{pdot}\hat{p}_{c,t} + \theta_y \hat{y}_t) + \varepsilon_{i,t} \tag{13}$$

The monetary authorities respond to deviations from target inflation and output with changes to nominal interest rates. $\varepsilon_i$ is the monetary policy shock. The parameters $\theta_{rg}$ represents the degree interest-rate smoothing. $\theta_{pdot}$ is the monetary response to inflation. $\theta_y$ denotes the monetary response to output.

- **World market**
The model assumes the UK is a small open economy and firms are price takers of world energy. Thus, the endogenous prices of energy are given by:

\[
\hat{p}_{o,t} = \varepsilon_{p_{o,t}} + \hat{s}_t
\]

(14)

\[
\hat{p}_{g,t} = \varepsilon_{p_{g,t}} + \hat{s}_t
\]

(15)

\(P_o\) denotes world prices of oil and \(P_g\) represents world prices of gas. \(\varepsilon_{p_{o}}\) and \(\varepsilon_{g}\) denotes the exogenous shocks of oil prices and gas prices, respectively.

Overall, the model comprises of forty-eight endogenous variables and twelve exogenous shocks. The twelve exogenous shocks follow a first-order autoregressive process, AR(1):

\[
\varepsilon_t = \rho \varepsilon_{t-1} + \eta_t
\]

(16)

where \(\rho\) denotes the shock’s persistence. \(\eta\) denotes the shock’s innovation and is identically independently distributed.

3.0 Data and Methodology

3.1 Data

The data sample is from 1981:Q1 to 2013:Q1, directed towards covering the great moderation period. Datastream is the main source of data. The study follows (Harrison et al., 2005) the Bank of England quarterly model to construct some of its data, like intermediate imports and employment hours. To be consistent with the model and literature, first, the data set were adjusted for inflation, with consumer price index used as proxy. Secondly, the data set is divided by working population to fit the stylized facts. Thirdly, take natural logarithm of the data set are taken, as the model is log-linearized, except for percentage rates which were divided by 100. Lastly, the data is detrended by Hodrick-Prescott filter with a smoothing parameter set at \(\lambda = 1600\). However, rates were detrended by removing their linear trends using spatial econometrics toolbox in Matlab.

Aggregate intermediate good is proxied with gross value added. The gross domestic product data is used for gross output. The oil and gas extraction sector (mining and quarrying) were subtracted from the volume of final output to obtain the non-energy gross output. Aggregate consumption is constructed by combining final household consumption expenditure and Non-profit institutions. Household energy consumption was collected separately. Following Schorfheide (2008) marginal cost is taken the labour share of gross output data. Real wage is constructed by dividing the average weekly earnings with consumption deflator. Data of energy input is constructed by combining gas and oil sale to industries, without double counting. Ninety days Treasury bill rate is taken the nominal interest rate while bank rate is proxied for rental rate. All rates are collected without further modifications. Finally, world prices of crude
oil and gas are collected in US dollars then converted to Pound Sterling using Pound to Sterling exchange rate data. In all, twenty-six variables were constructed.

3.2 Methodology - Indirect Inference Test
The study used the indirect inference (II) testing method to econometrically test that the model parameters took the model as close as possible to the data. The method follows Le, Meenagh and Minford (2017)\(^6\) to exactly identify the model parameters\(^7\). The actual data was compared with the simulated data using an auxiliary model. With filtered data, and the model residuals being stationary, the study took vector autoregression (VAR) as its auxiliary model. The test depends on the VAR coefficients and variances of the variables. The VAR model is given as:

$$
\begin{bmatrix}
    x_{1t} \\
    x_{2t} \\
    x_{3t}
\end{bmatrix} =
\begin{bmatrix}
    \beta_{11} & \beta_{21} & \beta_{31} \\
    \beta_{12} & \beta_{22} & \beta_{32} \\
    \beta_{13} & \beta_{23} & \beta_{33}
\end{bmatrix}
\begin{bmatrix}
    x_{1t-1} \\
    x_{2t-1} \\
    x_{3t-1}
\end{bmatrix} + \Omega_t
$$

The above VAR equation allowed joint distribution of three selected macroeconomic variables, \(x_{it}\), to explain the model. \(\Omega\) was the contemporaneous covariance matrix of the disturbance and the \(\beta\)'s are the coefficients of the model. When testing, variances of the actual data and the simulated data were added to give the dynamic model some volatility. The Matlab application was used for numerical computing. The Dynare path was used to obtain the first order approximation of the model.

3.3 Wald Steps for Calculating the Wald Statistic
In testing, first, the residuals from the structural model is collected. The residuals come from actual data applied on variables in the structural equations (Figure 2). For example, from (13), exchange rate is subtracted from oil prices to obtain oil prices shock. The data residuals are on the vertical axis and the sample period on the horizontal axis. With filtered data, the residuals display a normal distribution, as expected. Foreign residuals which includes energy prices residuals are quite high compared to others. Energy prices have been volatile while demand for exports declined. The residuals display higher activity during the Great recession. However, monetary policy shock is the exception due to lower interest rates and output.

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\(^{6}\) The method was efficient because, in estimation, it exactly identified the model parameters.

\(^{7}\) The estimated parameters are provided in the appendix.
The procedure satisfies that the number of residuals is less\(^8\) than the dependant variables in our model. The assumption that errors are independently and identically distributed following an AR(1) is also satisfied. This is estimated by ordinary least squares (OLS) method for the twelve residuals. An instrumental variable is run to estimate the equations that have expectations in them. For example, in (1), where the expected values of consumption in \(t + 1\) is to be estimated. A separate estimation taken to obtain the expected data. The data is then added to the structural equation to collect the residuals.

The next step is estimating the simulated data. The residual errors are the shocks' innovations\(^9\) while the coefficients are the shocks' persistence. Using Monte Carlo simulation, the innovations are drawn. This is done to ensure there is no simultaneity between the twelve shocks. The procedure produces 1000 bootstraps as independent samples of the simulated data. VAR of order 1, VAR (1), is applied to estimate the auxiliary models. VAR(1) captures the linear relationship of the variables in the joint distribution. From the estimates the VAR coefficients are collected. The variances of the actual data and the simulated data are then added to the distribution. Next, the covariance matrix \(W(\theta_0)\) of the distribution from the simulated data is estimated. The result gives a set of vectors, \(a_j (j = 1 \ldots N)\), which points out the sampling data variations as implied by the model:

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\(^8\) Number of residuals could also be equal to the number of endogenous variables.
\(^9\) Figure 8 depicts the imputed shocks.
\[ W(\theta_0) = \frac{1}{N} \sum_{j=1}^{N} (a_j - \bar{a}_j)'(a_j - \bar{a}_j) \]  

(17)

where \( \bar{a}_j = \frac{1}{N} \sum_{j=1}^{N} a_j \).

A single Wald statistic is estimated for the actual data and a 1000 Wald statistic for the bootstrapped simulated data. The Wald statistic \((WS)\) is defined as:

\[ WS = \left( g(a_T) - \overline{g(a_S(\theta_0))} \right)' W^{-1}(\theta_0) \left( g(a_T) - \overline{g(a_S(\theta_0))} \right) \]

(18)

where \( g(a_T) \) denotes the estimator for the simulated data. \( g(a_S(\theta_0)) \) denotes the estimator of the simulated data after estimating the mean of the distribution. \( W(\theta_0) \) denotes the variance-covariance of the simulated distribution which is obtained from the asymptotic distribution \( g(a_S) - \overline{g(a_S(\theta_0))} \). The asymptotic distribution is chi-squared. The Wald statistic of the actual data is compared to the simulated data’s Wald statistic to see if it falls within a significant level in the distribution. The exercise evident as the model is evaluated.

**4.0 Model Evaluation**

**4.1 Testing the Estimated Model**

The model was tested based on the Popperian principle, that the auxiliary model is wrong. Hence, the null hypothesis \((H_0)\) states that the model does not fit the data. Conversely, the alternative hypothesis \((H_1)\) states the model fits the data. A statistical comparison of the VAR coefficients provided a strong argument for the structural model to match the data. By applying VAR(1) to the joint distribution, nine elements were obtained as the VAR coefficients. The variances of the variables were then added to the joint distribution to obtain twelve elements in all. The selected variables in the joint distribution were gross output, annual inflation rate and nominal interest rate. Table 1 provides a summary of the test.

| Table 1 Summary of the Estimated Model’s Test to fit the Data. |
|-----------------|-----------------|-----------------|-----------------|
| Model dynamics | Mahalanobis distance | W/S | Wald percentile | p-value |
| Model dynamics and volatility | 0.7515 | 16.995 | 82% | 0.18 |
| Model volatility | 1.9378 | 9.247 | 97% | 0.03 |

The model is tested in three ways. Initially, the model dynamics is tested against the actual data. The dynamics of the model represents the VAR model’s transition overtime. If this test fails, there is no point going ahead. Then the variances are added to the model dynamics, the model volatility. Finally, the actual data variances are tested against the simulated data’s variances. In testing, the mahalanobis distance is used. The p-value is obtained by \( 1 - WS \).

The study rejected \( H_0 \) in the first two instances and conclude that the model fits the data.
Testing the model’s volatility alone means it only rejected $H_0$ at 1 percent significant level. Therefore, it rejected the null hypothesis and concluded that the dynamic model, combined with the model’s volatility, fit the data.

Table 2 Comparing the Actual VAR Coefficients Within the Simulated Model’s Boundaries.

<table>
<thead>
<tr>
<th>Joint distribution</th>
<th>Lower bounds</th>
<th>Upper bounds</th>
<th>Actual</th>
<th>IN/OUT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_y^y$</td>
<td>0.75627</td>
<td>0.98368</td>
<td>0.93392</td>
<td>IN</td>
</tr>
<tr>
<td>$A_\pi^y$</td>
<td>-0.16513</td>
<td>0.06595</td>
<td>-0.05477</td>
<td>IN</td>
</tr>
<tr>
<td>$A_r^y$</td>
<td>-0.08857</td>
<td>0.07659</td>
<td>-0.06204</td>
<td>IN</td>
</tr>
<tr>
<td>$A_\pi^\pi$</td>
<td>-0.03077</td>
<td>0.19167</td>
<td>0.10708</td>
<td>IN</td>
</tr>
<tr>
<td>$A_r^\pi$</td>
<td>0.72737</td>
<td>0.93325</td>
<td>0.81084</td>
<td>IN</td>
</tr>
<tr>
<td>$A_r^r$</td>
<td>-0.17663</td>
<td>0.01144</td>
<td>-0.09355</td>
<td>IN</td>
</tr>
<tr>
<td>$A_r^\gamma$</td>
<td>-0.00494</td>
<td>0.30348</td>
<td>0.15103</td>
<td>IN</td>
</tr>
<tr>
<td>$A_\pi^\pi$</td>
<td>-0.05853</td>
<td>0.26002</td>
<td>0.19083</td>
<td>IN</td>
</tr>
<tr>
<td>$A_r^\gamma$</td>
<td>0.69737</td>
<td>0.92347</td>
<td>0.73506</td>
<td>IN</td>
</tr>
<tr>
<td>$\sigma_y^2$</td>
<td>0.00004</td>
<td>0.00007</td>
<td>0.00003</td>
<td>OUT</td>
</tr>
<tr>
<td>$\sigma_\pi^2$</td>
<td>0.00003</td>
<td>0.00006</td>
<td>0.00003</td>
<td>IN</td>
</tr>
<tr>
<td>$\sigma_r^2$</td>
<td>0.00006</td>
<td>0.00011</td>
<td>0.00007</td>
<td>IN</td>
</tr>
</tbody>
</table>

Table 2 compares VAR coefficients of the actual data with the simulated data. The Wald percentile required the coefficients of the actual data to be within the 95 percent boundary. To achieve this, the study ranked the coefficients. Since the VAR coefficients of the simulated model came from 1000 simulations, the 25th column was chosen as the lower percentile and the 975th column as the upper percentile of the distribution. The set-up represented 2.5 percent of the upper boundary and 2.5 percent of the lower boundary in the distribution. The actual data coefficients were then compared to see if they fell within the distribution. All coefficients fall within the required percentile except the output variance. This is acceptable since the null hypothesis of joint distribution test is rejected. This empirically showed that the estimated model brought the model parameters closer to the data.

5.0 The Consequences of Shocks in the Model

5.1 Impulse Response Functions

The policymaker is concerned with the predictions of macroeconomic variables with respect to shocks. The study analysed the effects of two shocks on the model - a productivity shock and a world oil price shock – to provide an understanding of how the model worked in terms of the impulse response functions (IRF). These shocks independently cause economic fluctuations (Fouquet and van de Ven, 2017; Plosser, 1989). The key variables considered were gross output, aggregate consumption, nominal interest rate, annual inflation rate, real wages,
exchange rate, energy use and employment hours. Each shock’s standard deviation shock was estimated from the structural errors collected using actual data. This is shown using a graphical image, which showed how long the shocks lasted in quarters. For all aftershock effects, the horizontal axis denoted the period in quarters. Forty quarters were used (as was found to be standard in previous literature) to analyse the effects following each shock. The decimal points on the vertical axis represented changes, except for the effects on interest rate which were converted to basis points.

Figure 3 shows how the key variables will respond to a standard deviation (1%) shock of productivity in the economy. The model predicts gross output increases by 0.5% and oscillates for about twelve quarters (three years) after the shock before converging. Consumption is predicted to rise by 0.3% because real wages have increased, which made households wealthier. Employment in the economy will rise for only five quarters. This meant that income effects dominated as consumption took more than forty quarters to converge. For firms, energy use increases as gross output rises. The effect will last for about twelve quarters, like gross output’s effect. This is due to sticky prices, as input demand did not respond to the rise in gross output immediately. Since the shocks are temporary, the authorities will respond with lower interest rates to clear the goods market and to stimulate investment as households are wealthier.
Figure 4 shows how the selected variables will respond to one standard deviation shock in world oil prices. This means a rise in oil prices. For any rise in oil prices, firms will pass it on to the consumer as quickly as possible. The model predicts firm’s energy demand will decline for about 15 quarters. As energy is a complimentary input without a substitute, the demand recovers quickly. Firms respond quickly by increasing output prices to reach potential output, which is why the fall would be small. Such responses predicts inflationary pressures and, hence, a fall in real wages. The exchange rate will appreciate because world prices rose relative to domestic prices. Output is predicted to decline together with employment hours and real wages. This causes a decline in consumption, which is predicted to take more than forty quarters to converge. The economic welfare of the UK declines with a positive world energy price shock due to a decline in output and consumption. These findings are consistent with the literature (Anciaes et al., 2012; Baffes et al., 2015; Berument et al., 2010; Difiglio, 2014). Authorities respond by more borrowing for the shortfall in gross output. Thus, there is a rise in nominal interest rates and inflation in the economy. This is deemed to be an unfavourable shock.

The responses of the variables to the oil price shock are qualitatively similar to those of the gas price shock. Overall, it can be stated that the energy price shock caused a decline in gross output where the monetary authorities’ intervention compensated for the decline.

### 5.2 VAR-IRFs

A check with VAR bounds was run to predict the model’s IRFs (Figures 5 and 6). The policymaker is mainly concerned with the model’s IRFs. The VAR-IRFs of the estimated

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10 The vertical axis and horizontal axis are consistent with the IRF explanation, provided in section 5.1 above.
model are plotted using gas price shock and monetary policy shock. The test was based on the VAR(1) and the coefficients\(^{11}\) of the simulated data. The starting point of the model’s IRFs was used as the starting point, and the simulated data coefficients were ranked to allocate the 2.5 percent upper boundaries and 2.5 percent lower boundaries. The process generated 95 percent confidence limits for the implied VAR responses that simply included the data-based VAR responses to the structural shocks for the variables in the auxiliary model - gross output, inflation and interest rate.

A one standard deviation point shock of gas prices makes output to fall and inflation to rise in the economy. The key variables appear within the statistical bounds. Comparatively, gas price shock is predicted to have higher effect on output decline than oil price shock (in Figure 4). Output declines for fifteen quarters following one gas prices shock to the economy. The monetary authorities responded by increasing nominal rates and borrowing to cover for the output loss. Such response creates inflationary pressures in the economy. Figure 6 shows the predicted monetary policy responses as a shock. It shows one standard deviation monetary policy shock will increase both the base rate and the output level, as well as create inflation in the economy.

5.3 A Stochastic Variance Decomposition
The study presented the robust results of the simulated data to understand the significance of each shock with respect to the variability of the key endogenous variables. Table 3 presents the

\(^{11}\) These are represented by the red dotted lines, indicating 95 percent confidence intervals for the point estimates.
shocks horizontally and the key variables vertically, and all results are presented using percentages. The study combined some shocks to show the domestic demand shocks and the foreign demand shocks.

Table 3: Variance Decomposition of Implied Shocks.

<table>
<thead>
<tr>
<th></th>
<th>Productivity</th>
<th>Monetary Policy</th>
<th>Price mark-up</th>
<th>Domestic demand</th>
<th>Foreign demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>1.47</td>
<td>34.21</td>
<td>1.51</td>
<td>47.45</td>
<td>15.36</td>
</tr>
<tr>
<td>Energy use</td>
<td>0.88</td>
<td>34.79</td>
<td>15.74</td>
<td>34.83</td>
<td>13.76</td>
</tr>
<tr>
<td>Inflation rate</td>
<td>0.58</td>
<td>63.97</td>
<td>0.67</td>
<td>32.17</td>
<td>2.6</td>
</tr>
<tr>
<td>Nominal interest rate</td>
<td>0.57</td>
<td>9.5</td>
<td>0.24</td>
<td>82.38</td>
<td>7.31</td>
</tr>
<tr>
<td>Exchange rate</td>
<td>0.16</td>
<td>40.03</td>
<td>0.1</td>
<td>33.91</td>
<td>25.8</td>
</tr>
<tr>
<td>Real wages</td>
<td>3.96</td>
<td>16.52</td>
<td>51.63</td>
<td>19.88</td>
<td>8.02</td>
</tr>
</tbody>
</table>

Productivity shocks accounted for 21 percent of gross output variability. Foreign demand, which included energy prices, explains 13 percent of gross output variability as intermediate imports counted in the bundle of inputs in the production function. The intermediate imports include energy commodities. Given that the UK was a net oil importer, such values could explain the significance of energy in economic production. Monetary policy played a significant role in gross output as it explained over 30 percent of its variability. Consumption variability was dominated by domestic demand and monetary policy at a combined rate of 82 percent. However, foreign demand shocks and productivity are significant in explaining aggregate consumption as UK households engaged in international trade, including buying energy.

For energy use, as a complimentary production input, productivity shock explained about 1 percent. Monetary policy and price mark-up explanation came in the form of subsidies provided for prices to remain unchanged. Monetary policy dominated the inflation rate in the economy as it set the borrowing rates in the economy. Consumption variability was dominated by domestic demand and monetary policy at a combined rate of 82 percent. Consumption decisions and the trade-off between consumption and leisure is the reason why domestic demand shocks dominated the nominal interest rate. The exchange rate was explained by the foreign demand shocks, mostly by the foreign interest shock. The monetary policy shock

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12 This is a common practice in literature (see Smets and Wouters, 2007 and Aminu, et al., 2018).
13 Domestic demand shocks were a combination of preference shocks, investment-specific technology, wage mark-ups and government expenditure shocks.
14 Foreign demand shocks were energy price shocks, export shocks, foreign interest-rate shocks and import price shocks.
dominated as it did for domestic inflation. Price mark-ups explained real wages by over 50 percent, which was consistent with previous literature on bargain models (Sanfey, 1998; Gali, Gertler and Lopez-Salido, 2007).

Unsurprisingly, domestic demand and monetary policy shocks dominated the contribution of key macroeconomic aggregates in the economy. The monetary policy shock contributions were consistent with the New-Keynesian assumption that monetary policies have a strong influence on economic activity. In contrast, Aminu (2018), found foreign shocks and productivity shock have little significance on key variables. This is explained by lower calibration values assigned to the model steady-state parameters.

### 5.4 Shocks Impact During the Great Recession

The study compared the behaviour of each shock in the Great Moderation period (sample period) and the Great Recession period (crisis episode). This analysis shed light on the impact of exogenous shocks. The study found changes in the volatility and persistence of key shocks.

<table>
<thead>
<tr>
<th>Shock (j)</th>
<th>$\rho_j$ Sample period</th>
<th>$\rho_j$ Crisis episode</th>
<th>Change %</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity shock</td>
<td>0.6453</td>
<td>0.5499</td>
<td>-14.8</td>
<td>Low</td>
</tr>
<tr>
<td>Preference shock</td>
<td>0.8816</td>
<td>0.9127</td>
<td>3.5</td>
<td>High</td>
</tr>
<tr>
<td>Government spending shock</td>
<td>0.7811</td>
<td>0.7617</td>
<td>-2.5</td>
<td>Low</td>
</tr>
<tr>
<td>Monetary policy shock</td>
<td>0.8946</td>
<td>0.8415</td>
<td>-5.9</td>
<td>Low</td>
</tr>
<tr>
<td>Investment-specific technology shock</td>
<td>0.8172</td>
<td>0.7446</td>
<td>-8.9</td>
<td>Low</td>
</tr>
<tr>
<td>Price mark-up shock</td>
<td>0.5823</td>
<td>0.1490</td>
<td>-74.4</td>
<td>Low</td>
</tr>
<tr>
<td>Gas price shock</td>
<td>0.8701</td>
<td>0.7512</td>
<td>-13.7</td>
<td>Low</td>
</tr>
<tr>
<td>Imports price shock</td>
<td>0.9536</td>
<td>0.9305</td>
<td>-2.4</td>
<td>Low</td>
</tr>
<tr>
<td>Oil price shock</td>
<td>0.7944</td>
<td>0.7394</td>
<td>-6.9</td>
<td>Low</td>
</tr>
<tr>
<td>World interest rate shock</td>
<td>0.8385</td>
<td>0.8407</td>
<td>0.3</td>
<td>High</td>
</tr>
<tr>
<td>Wage mark-up shock</td>
<td>0.9383</td>
<td>0.8981</td>
<td>-4.3</td>
<td>Low</td>
</tr>
<tr>
<td>Exports shock</td>
<td>0.9328</td>
<td>0.8704</td>
<td>-6.7</td>
<td>Low</td>
</tr>
</tbody>
</table>

Table 4 shows the shock’s persistence. The existence of shock persistence implies that there was a connection between the current period and past periods. Theoretically, long term persistence should have a higher frequency than short term persistence. As business cycles happen in the short term, persistence should be low. With low productivity during the Great Recession, output growth should depend less on the last period. This means economic policy and energy prices can influence short-term output growth. Energy price shocks increased the contraction of output and economic activity. The high persistence of preference shock is

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15 This is evident in Figure 8.
related to UK financial intermediaries’ behaviour during the crisis. They induced the crisis by rationing lending to households. Despite the efforts made by monetary authorities, consumption habits took time to adjust. The equal response of world economies to interest rates made capital movement less attractive. Therefore, the world interest rate persistence was high.

Table 4 Shock’s Volatility.

<table>
<thead>
<tr>
<th>Shock (j)</th>
<th>$\sigma_j$ Sample period</th>
<th>$\sigma_j$ Crisis episode</th>
<th>Change %</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Productivity shock</td>
<td>0.0106</td>
<td>0.0132</td>
<td>25</td>
<td>High</td>
</tr>
<tr>
<td>Preference shock</td>
<td>0.0142</td>
<td>0.0163</td>
<td>15</td>
<td>High</td>
</tr>
<tr>
<td>Government spending shock</td>
<td>0.0111</td>
<td>0.0143</td>
<td>30</td>
<td>High</td>
</tr>
<tr>
<td>Monetary policy shock</td>
<td>0.0128</td>
<td>0.0116</td>
<td>-9</td>
<td>Low</td>
</tr>
<tr>
<td>Investment-specific technology shock</td>
<td>0.0110</td>
<td>0.0147</td>
<td>33</td>
<td>High</td>
</tr>
<tr>
<td>Price mark-up shock</td>
<td>0.0038</td>
<td>0.0065</td>
<td>72</td>
<td>High</td>
</tr>
<tr>
<td>Gas price shock</td>
<td>0.0744</td>
<td>0.1154</td>
<td>55</td>
<td>High</td>
</tr>
<tr>
<td>Imports price shock</td>
<td>0.0233</td>
<td>0.0279</td>
<td>20</td>
<td>High</td>
</tr>
<tr>
<td>Oil price shock</td>
<td>0.1265</td>
<td>0.1360</td>
<td>8</td>
<td>High</td>
</tr>
<tr>
<td>World interest rate shock</td>
<td>0.0147</td>
<td>0.0213</td>
<td>45</td>
<td>High</td>
</tr>
<tr>
<td>Wage mark-up shock</td>
<td>0.1375</td>
<td>0.1334</td>
<td>-3</td>
<td>Low</td>
</tr>
<tr>
<td>Exports shock</td>
<td>0.0826</td>
<td>0.0975</td>
<td>18</td>
<td>High</td>
</tr>
</tbody>
</table>

Table 5 shows the volatility of each shock. The short term volatility was higher, except in terms of wage mark-up and monetary policy shocks. The frequency of productivity shock was twenty-five percent higher than that found in the sample period. This explains the effort made by firms to adjust to the economic activity. Gas prices were highly volatile, at fifty-five percent. Figure 7 shows that gas prices increased by one hundred and twenty percent between 2007:Q4 and 2008:Q4 and that oil prices increased by sixty-six percent between 2007:Q3 and 2008:Q3. As firms looked to pass over the high costs of energy prices, inflationary pressures arose. Thus, price mark-up volatility rose to seventy-one percent. The wage mark-up compared with households’ ability to set real wages was only possible by generating wage inflation as consistent with theory (Gali, 2011). Monetary policy shocks had lower volatility due unconventional monetary policies.

5.5 What Energy Shocks Say About the Model

The study analyses how energy prices influence economic activities by decomposing the shocks during Great Recession. It reports the contribution of productivity shock, monetary policy, energy prices, domestic demand and foreign demand on gross output growth.
The peak of gross output decline was experienced in the last quarter of 2008 (Figure 7) during the Great Recession. However, the start of 2008 was met by rising energy prices (see Figure 9). The mixed reaction of oil and gas prices in 2008:Q4 made the energy shocks contribution to be smaller in that quarter. The increase in energy prices are passed on to final goods quickly. Given energy prices rose faster than average, we can see higher contribution of price mark-up. Domestic demand contributes to gross output decline as household consumption decline. This is due to lack of lending by financial intermediaries. From third quarter of 2008, the Bank of England decreased lending rates and engaged in some unconventional monetary policies. Conclusively, energy prices are significant in driving economic activities while monetary policy has been used to overcome its consequences.

6.0 Conclusion

In summary, this study used an estimated DSGE model of energy in the UK. The features of the model were those of the NK model with a wide range of nominal and real frictions. The model identified energy supply chains in the economy. The model was able to explain the consequences and effects of volatile energy prices on the UK economy. With energy as a production input, the study found that firms could only pass the cost of higher energy prices to consumers. This makes demand for output to decline. Firms that produced non-energy output optimised by making changes to the components of input costs. Such behaviour created inflationary pressures in the economy. The high volatility of energy prices was explained as a trend increase. Any rise in energy prices would reduce economic welfare as aggregate output
and consumption declined. The results showed the importance of energy price shocks and how they drive the business cycle. Using shock decomposition, the study showed that energy price shocks amplified output contraction during the Great Recession.

Since the shocks were temporary, the monetary authorities borrowed to cover output loss given the shock to the economy was temporary. The low volatility of monetary policy shocks indicated the actions taken during the Great Recession to accelerate the economy out of crisis. These include lowering nominal interest rate to 0.5 percent and quantitative easing by asset purchase. The sustainable investment rule justified policymakers’ actions. Therefore, monetary policy was used to off-set the consequences of volatile energy prices. The policy implication reconciles with both Hamilton and Herrara (2004) and Bernanke et al. (1997) that monetary policy can off-set volatile energy prices albeit unconventionally. This study could be further applied to energy exporting economy to study the impact of energy prices volatility.


Appendix

Simulated Annealing Estimates of the model

Table 6 values of estimated parameters with the UK data from 1981:Q1 to 2013:Q1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi_u$</td>
<td>Utility firm’s probability of not changing the price</td>
<td>0.5288</td>
</tr>
<tr>
<td>$\chi_{pp}$</td>
<td>Petroleum firms probability of not changing the price</td>
<td>0.7679</td>
</tr>
<tr>
<td>$\chi_p$</td>
<td>Non-energy firm’s probability of not changing the price</td>
<td>0.546</td>
</tr>
<tr>
<td>$\varepsilon_u$</td>
<td>Utility firm’s degree of indexation</td>
<td>0.3247</td>
</tr>
<tr>
<td>$\varepsilon_{pp}$</td>
<td>Petroleum firm’s degree of indexation</td>
<td>0.8101</td>
</tr>
<tr>
<td>$\theta_{rg}$</td>
<td>Monetary rule’s degree of interest-rate smoothing</td>
<td>0.3048</td>
</tr>
<tr>
<td>$\theta_y$</td>
<td>Monetary rule’s coefficient on output</td>
<td>0.1473</td>
</tr>
<tr>
<td>$\theta_{pdot}$</td>
<td>Monetary rule’s coefficient on inflation</td>
<td>2.5441</td>
</tr>
<tr>
<td>$\varepsilon_p$</td>
<td>Non-energy firm’s degree of indexation</td>
<td>0.4707</td>
</tr>
<tr>
<td>$\varepsilon_{pm}$</td>
<td>Imports degree of indexation</td>
<td>0.5841</td>
</tr>
<tr>
<td>$\xi_{pm}$</td>
<td>Probability of not able to change price: importers</td>
<td>0.161</td>
</tr>
<tr>
<td>$\eta_x$</td>
<td>World demand’s elasticity</td>
<td>3.4465</td>
</tr>
<tr>
<td>$\psi_z$</td>
<td>World demand’s degree of persistence</td>
<td>0.2</td>
</tr>
<tr>
<td>$\psi_{hab}$</td>
<td>Household’s degree of habit formation</td>
<td>0.5135</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Intertemporal elasticity of substitution</td>
<td>0.7209</td>
</tr>
<tr>
<td>$\phi_z$</td>
<td>Inverse elasticity of capital utilisation costs</td>
<td>0.6613</td>
</tr>
<tr>
<td>$\varepsilon_k$</td>
<td>Degree of persistence of adjustment cost of investment</td>
<td>0.7071</td>
</tr>
<tr>
<td>$\chi_z$</td>
<td>Scale of investment adjustment cost</td>
<td>4.4701</td>
</tr>
<tr>
<td>$\xi_w$</td>
<td>Wage indexation degree</td>
<td>0.9631</td>
</tr>
<tr>
<td>$\sigma_h$</td>
<td>Frisch elasticity of labour supply</td>
<td>0.0102</td>
</tr>
<tr>
<td>$\psi_{wc}$</td>
<td>Share of wage bill paid financed by borrowing</td>
<td>0.4493</td>
</tr>
<tr>
<td>$\psi_w$</td>
<td>Probability of being able to change wages</td>
<td>0.4222</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>Elasticity of demand for differentiated labour</td>
<td>1.1</td>
</tr>
</tbody>
</table>
Figure 8 Imputed Shocks

Figure 9 World Energy Prices (2007:Q1 – 2011:Q4).