Sonification of Controlled Quantum Dynamics

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ABSTRACT
In the present paper we employ sonification and simple electroacoustic composition techniques to represent simulated controlled quantum dynamics. We suggest an intuitive sonification process in order to represent acoustically and musically an important quantum phenomenon that is used in quantum computation. A interesting problem in this field has been to understand why states float back and forth between a number of configurations, seemingly unguided, and yet almost miraculously reach target eventually. Through Parameter-Mapping-Sonification we explore the dynamics of this microscopic peculiar system. In our current research, our sonification choices have both a functional and an aesthetic goal.

1. INTRODUCTION
Music and Quantum Dynamics share a wave-like nature. Many analogies and surprising connections between the two fields exists, which can be illustrated and explored through sonification techniques [1]. In the presented project, compositional and sonification techniques have been employed to represent simulated controlled quantum dynamics [2].
Sonification, even if it is a quite young field of research, has been used extensively in the last 25 years by scientists and artists. Many definitions and interpretations have been given to it, probably the most rigorous being presented in [3]. Lately a lot of attention has been paid to it because of the application of sonification techniques to the newly discovered Higgs Boson by the pan-European GÉANT research network who turned these findings into music [4].
As we will see in the following section, very little research has been done in the area of sonification of quantum mechanical systems. Particularly the sonification of numerical simulations of controlled quantum dynamics are explored for the first time according to the authors knowledge. A long-standing problem in quantum mechanics has been to understand why states float back and forth between a number of configurations, seemingly unguided, and yet almost miraculously reach target eventually. Our thesis is that this is caused by competing frequency scales which eventually interfere in harmony. Transferring methods from sonification and music theory therefore could lead to novel techniques- this is substantiated by preliminary results. The current project is an ongoing research. The final original sonifications presented in the paper are approached as conveyors of information and as artifacts with aesthetic value.
The paper is organised into four sections. Section two offers some background work related to the field of sonification. In section three we focus more on the quantum mechanical background of our research. Our sonification design and explorations are presented on section four and a presentation of the technical aspects of the project are briefly described in section five. Finally section six offers a discussion and an informal evaluation of our approach.

2. CONTEXT AND PREVIOUS WORK
Data sonification is a relatively recent promising emerging research field that examines how the human auditory system can be used to better understand changes and structures coming from any data streams [1]. In the field of physics it has been used since the geiger counter, a detector for nuclear radiation, in the early 20th century. Several works related to the sonification of quantum-mechanical phenomena already exist. Some of them clearly lay on the boarder of scientific research and artistic exploration.
On the purely scientific frontier, sonification has been used in quantum mechanics in order to display scientific results. Numerical data from the simulation of the XY-spin model have been sonified using phase modulation [5]. An investigation on the sonification of quantum spectra has been explored in [6]. In this work, sonification has been used as tool for the classification and the exploration of baryon properties by mapping the mass spectra to frequency spectra or to a scalable pitch range. The quantum harmonic oscillator has been studied by Saranti, Eckel and Piro [7]. Eigenenergies of the system were projected on the frequency plane as frequencies of sinusoids, and simple audification procedures have been employed for the time-dependent perturbation. A metaphor procedure has been suggested in [8], which approached sonification from an inclusive design perspective.
From a purely musical point of view Bain has created a simple algorithmic composition process and a composition based on the Schrödinger equation [9]. A similar physical system has been explored by Fischman [10] for the generation of musical material which are associated both on
the structure and the sonic realization of the composition by asynchronous granular synthesis techniques. Sturm has developed a technique of a sound-based composition using quantum mechanics by employing both scientific and compositional concerns [11]. Another research project and musical composition has been created by Delatour [12], by concentrating on the molecular vibrational spectra data.

It is also interesting to mention at this point the essay of Lyndon Stone related to the representation of the phenomenon of quantum superposition represented by contemporary visual artists [13]. Voss-Andreae, both trained as a physicist and artist like the first author of the current paper, has also created a body of sculptural work related to quantum concepts [14]. In our research and artist endeavor it is important to understand how visual artists represent and engage with quantum mechanical ideas since we are simultaneously working on a music composition and an audiovisual installation.

3. CONTROLLED QUANTUM DYNAMICS

In this section we describe the basic quantum mechanical background to the project. A more detailed presentation can be found in the introductory textbook [2]. Over the last few decades it was discovered that the seemingly incomprehensible and weird dynamics of nanoscopic objects - quanta - can actually be used to perform computations. This in itself should not surprise, as any sufficiently rich dynamics can be used for computation (even a pool table [15]). The amazing result however was that for certain tasks, computation with quanta could be exponentially faster than any classical machine. As a consequence, quantum computers can find solutions to problems which are practically unsolvable on our current machines.

There is a large interest in building these novel type of computers. Roughly speaking, the implementation of a quantum algorithm \( U(T) \) in a \( d \)-dimensional quantum system corresponds to finding a time-dependent function \( f(t) \) on the time interval \([0, T]\) such that the solution to the first order differential equation

\[
\frac{d}{dt} U(t) = (H_0 + f(t)H_1)U(t) \tag{1}
\]

is given by \( U(T) \). Here, \( U(t) \), \( H_0 \) and \( H_1 \) are \( d \times d \) matrices with complex entries, and the initial condition of the differential equation is the identity matrix \( U(0) = I \). The function \( f(t) \) corresponds to a control pulse which is applied to the system; \( H_0 \) describes the dynamics of the system in the absence of such control; and \( H_1 \) describes how our control interplays with the dynamics.

It might be helpful to provide a specific example of a system which would be describable by the above equation (also known as Schrödinger’s equation). Consider an electron which can take \( d \) different positions on a one-dimensional lattice, and hop from one site to the other with the interaction

\[
H_0 = \begin{pmatrix}
1 & 1 & 0 & 0 & \ldots & 0 \\
1 & 1 & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \ddots & \ddots & \ddots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & 0 & 1 & 1
\end{pmatrix} \tag{2}
\]

We try to control the movement of the electron by changing the energy it experiences when it sits at the first site,

\[
H_1 = \begin{pmatrix}
1 & 0 & 0 & 0 & \ldots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \ddots & \ddots & \ddots & 0 \\
0 & 0 & \ddots & \ddots & \ddots & 0 \\
\vdots & \vdots & \ddots & \ddots & \ddots & \ddots \\
0 & 0 & \ldots & 0 & 0 & 0
\end{pmatrix} \tag{3}
\]

Although this looks like a very limited type of control, as it is only experienced when the electron is on the first site, it turns out that using combinations of \( H_1 \) with the natural evolution \( H_0 \), any quantum algorithm may be executed in the system[16]. The algorithm \( U(t) \) can be interpreted in the following way: \( |U_{nm}(t)|^2 \) corresponds to the probability of finding the electron at site \( n \) at time \( t \), provided it started at site \( m \) initially. This probability quickly delocalizes, meaning that the electron appears to be at several positions simultaneously (we will use this aspect later in the sonification).

The most puzzling aspect of controlled quantum dynamics is the way the system moves from the initial configuration to the final one in a very peculiar way. For instance, it seems to first go very close to the goal, but then move far away again, only to return later even more closely. This is a bit like the dynamics of a wave, and it hints to the aforementioned links between music and quantum theory. It has been a long-standing goal in control theory to develop a better understanding of this wave dynamics [17], and here we aim to do so with the tools of sonification.

4. SONIFICATION

Our sonification choices have both a functional and an aesthetic goal. The research (and the reached outcomes so far) forms the basis of musical compositions and scientific systematic investigation. Therefore according to the broad categorization suggested in [1], it covers both data exploration and the art-entertainment categories. What is more, it is interesting to mention that our sonification decisions have been influenced equally by artistic choices and by scientific dictation by both authors when that was possible. However, we should not forget what Sturm warned “Just as in physical modeling synthesis, physics is at the service of the composer creating innumerable possibilities. However, here the composer has become restricted by the mathematicians and methods of physics, and the scientist a slave to the aesthetics and methods of music. Thus in order to employ these techniques in a musically effective way one needs both scholarships.” [18].

The data we use in the present research are the elements of some selected columns of the matrix \( U(t) \). An example of a goal unitary matrix \( U_g \) is
The size of the matrix in most of the cases is $15 \times 15$ or $30 \times 30$, corresponding to sizes which are currently experimentally relevant. This process of sonification and artistic creation relies on the iterative process of listening, assessing, and refining the system architecture and on more formal approaches coming from quantum mechanics and music theory. Below we present some of our mapping decisions\(^1\). The critical analysis of the results will be presented on the last section of the paper.

### 4.1 Mapping column index to pitch / magnitude to amplitude

Each possible electron site is mapped to a scalable pitch range drawn from a music scale. The scale selection consists a design choice with an element of artistic freedom. In the present examples we have chosen a chromatic, a major and a harmonic minor scale. The magnitude of a single column of the matrix $U(t)$ is mapped to amplitude. Therefore the whole column appears as a chord. During the simulation different chords are heard in succession. The mapping can be expressed mathematically:

$$|U_{nm}(t)|^2 \rightarrow \text{amplitude}$$

$$n \rightarrow \text{pitch}$$

The result of this sonification is sequence of tunable chords. The selected scale defines at great extent the harmonic vocabulary of the result. Clearly the chromatic scale gives more an atonal character to the result with references to chromaticism \[19\]. On the other side the unmodulated tonal centre and the repetition has references to minimal music. The pulse is static in both cases.

When we employ the same process on two different rows and we pan them accordingly in a stereo layout, we perceive an interesting spatial movement. By using a multi-channel sound reproduction system the movement becomes more dramatic.

### 4.2 Mapping column index to pitch / phase to time delay

This sonification expands the previous one by using phase information. The most intuitive mapping was when the phase of a single row of each element of the matrix $U(t)$ is mapped to a time delay. The mapping can be expressed mathematically:

$$|U_{nm}(t)|^2 \rightarrow \text{amplitude}$$

$$n \rightarrow \text{pitch}$$

$$\angle U_{nm}(t) \rightarrow \text{delay}$$

\(1\) The sound files of the presented examples can be found at https://www.dropbox.com/sh/wtpazymrd77/p44qIN9PAWjWTT

The result of this sonification is sequence of tunable irregular arpeggios. This effect is directly related to the the phase relationships of the possible states of the particle. The rhythmic repetition breaks and interesting phasing patterns appear slowly.

### 4.3 Mapping column index and magnitude to additive synthesis parameters

An additive synthesizer has been designed in order to map the magnitude of a single row of each element of the matrix $U(t)$ is mapped to the amplitude of the partials. This mapping dissolves the strong rhythmical domination of the previous sonifications and suggests continuity. The resulted texture undulates according to the possible position of the electron. It can be expressed mathematically:

$$|U_{nm}(t)|^2 \rightarrow \text{amplitude}$$

$$n \rightarrow \text{partial}$$

This type of mapping evokes an alternative listening experience which is more subtle. Emphasis is putted on the smooth spectral development and the result is closer stylistically to ambient music.

## 5. TECHNICAL IMPLEMENTATION

As stated already earlier in the paper, the project has various objectives emerging from the scientific and the artistic components of the research. We have employed two programming environments in order to work and develop our numerical simulations and our sonification algorithms. We have also used a digital audio workstation in order to prototype faster some aspects of our project and for the post-production.

The main research aim concerning quantum simulations is the engineering of control functions for quantum dynamics. We employ the open source software package Dynamo, which implements a BFGS global optimization routine \[20\]. This method is modified by a set of algorithms developed by D. Burgarth which allow a dynamical creation of visualization data and focus on the complexity of the dynamics. A script written in Matlab language calculates the dynamics of the system a generates a plain text file with the simulated data. Every line of the file is numbered and consists of the elements (complex numbers) of the matrix $U(t)$ at the instant $t$.

The data were imported and reformatted in Max graphical programming environment where the actual sonification took place. Parameter Mapping Sonification Technique is used to associate the quantum information into auditory and musical parameters. The MIDI (Musical Instruments Digital Interface) protocol has been used in order to control and communicate with the digital audio workstation. In the basic setup, quantum variables and the control function are translated into MIDI control messages and MIDI note events. Those message are processed, smoothed, scaled and mapped to nonlinear transfer functions before transmitted to software sound synthesis engines our to external digital audio workstations. In the last case, the processed MIDI information is eventually mapped into parameters of
digital audio effects and commercial software synthesizers and samplers. The quantum data are mapped according to a simple transfer function into frequency/sound level and more precisely into MIDI (Musical Instruments Digital Interface) data that are controlling instruments (in the preliminary example, a piano software synthesizer).

6. RESULTS AND CONCLUSIONS

Even though the project is in a work-in-progress stage, we have achieved already some interesting and very encouraging results. From a physics perspective, we could clearly listen and perceive several interesting and relevant effects through the sonification. Probably the main advantage compared to visualisation is that we can tap into several rows of the time evolution simultaneously, which would be hard visually. A summary of the most interesting result is:

Correlations:

A quantity of high interest in quantum dynamics are correlation functions, which provide information on how likely it is to find the particle in one position given its probability to be at another site. These correlations are particularly high between nearest neighbors on the lattice. In the sonification, they correspond to a movement and echo between the stereo channels. Correlations functions are usually mathematically challenging to read, so it is useful to be able to hear them directly.

Interferences:

Quantum dynamics is a wave dynamics and therefore dominated by interference. In our sonification interference becomes very easy to discover, with positive (constructive) interference playing high velocity keys, and negative (destructive) interference resulting in silence. Raising and lowering of the pitch tells us in which direction the particle is moving, and we can identify reflections at the chain end through positive interferences.

Phase information:

A typical effect in wave dynamics is that the group velocity (roughly the speed at which the expected value of the particle move) is much slower than the phase velocity. This implies that the phase pattern of the wave is constantly overtaking it. In the sonification including phases, we can identify this effect easily and analyse it. We found that the audible dynamics strengthens the conjecture that the strange paths taken by controlled quantum dynamics is due to subtle interference effects.

In general, rendering sound and even developing algorithmic composition strategies in response to data like the one obtained from quantum systems can enhance the research in quantum mechanics and develop a new vocabulary in musical and audiovisual creation. Moreover, we believe that the application of concepts coming from the western art music theory such as harmony and tuning systems into quantum control may result into models with desired dynamic behavior.

For future work, we would like to simultaneously persuade research on the sonification the control function. We have already started experimenting with several mapping scenarios such is influencing the time evolution of the events or controlling a digital audio effect applied to the final audio outcome. Those investigations has not been proved useful yet put they revealed very interesting questions for the future. The authors have also already started working on an audiovisual art installation with many displays and a multichannel diffusion system. The visualisation strategy remains open but some preliminary results demonstrated the strength of having sonification and visualisation procedures simultaneously.

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7. REFERENCES


